AHOI Workshop on Tempo-Spatial Stochastic Processes and Stochastic Volatility

Imperial College London
23–24 February 2015

Organisers:
Almut Veraart
Mikko Pakkanen

The workshop is funded by A. Veraart's Marie Curie FP7 Integration Grant (within the 7th European Union Framework Programme).

Programme

Monday, 23 February 2015

Morning Session, 8:45–12:00. Chair: Almut Veraart

8:45–9:00	Welcome
9:00–9:35	Ivan Nourdin
9:40–10:15	Lasse Leskelä
10:15–10:45	*Coffee & Tea Break*
10:45–11:20	Andreas Basse-O'Connor
11:25–12:00	Anthony Réveillac

Lunch Break, 12:00–14:00.

Afternoon Session, 14:00–17:00. Chair: Mark Podolskij

14:00–14:35	Murad S. Taqqu
14:40–15:15	Markus Riedle
15:15–15:45	*Coffee & Tea Break*
15:45–16:20	Bezirgen Veliyev
16:25–17:00	Shin Kanaya

Poster Session, 17:15–18:45. Mathematics Common Room, 5th Floor of Huxley Building (180 Queen's Gate).

Conference Dinner at 19:00, Ognisko Restaurant (55 Prince's Gate, Exhibition Rd).

Tuesday, 24 February 2015

Morning Session, 9:30–11:55. Chair: Robert Stelzer

9:30–10:05	Mark Podolskij
10:10–10:40	*Coffee & Tea Break*
10:40–11:15	Anita Behme
11:20–11:55	Carsten Chong

Lunch Break, 12:00–13:45.

Afternoon Session, 13:45–16:45. Chair: Mikko Pakkanen

13:45–14:20	Heidar Eyjolfsson
14:25–15:00	Benedykt Szozda
15:00–15:30	*Coffee & Tea Break*
15:30–16:05	Robert Stelzer
16:10–16:45	Salvador Ortiz-Latorre

End of the Workshop, 16:45.

Special Event: A Seminar Talk by Prof Bernard Silverman FRS, 17:00–18:00. Clore Lecture Theatre, 2th Floor of Huxley Building (180 Queen's Gate), see page 13.

Information

Scope and Aims of the Workshop

The workshop brings together experts in probability theory and statistics to discuss recent advances in the stochastic modelling of tempo-spatial phenomena. Particular emphasis is placed on the modelling of stochastic volatility in time and space. In addition this workshop also aims at presenting some of the most recent developments concerning new statistical inference methods for tempo-spatial stochastic processes and random fields.

Venue

The workshop (including lunch and coffee breaks) takes place at *170 Queen's Gate* on the South Kensington Campus of Imperial College London (SW7 2AZ), *except for the Poster Session* (on Monday, 23 February at 17:15–18:45), which is held in the Mathematics Common Room, 5th Floor of *Huxley Building* at 180 Queen's Gate. See the map on page 15.

Conference Dinner

All invited speakers and poster presenters are invited to join us for the Conference Dinner at *Ognisko Restaurant* (55 Prince's Gate, Exhibition Road, London SW7 2PN) on Monday, 23 February at 19:00. See the map on page 15.

About the AHOI Network

AHOI (AarHus, Oslo, Imperial) is a collaborative network between researchers in Stochastics at *Aarhus University*, *University of Oslo* and *Imperial College London*. The purpose of the network is to foster basic research in the theory and applications of *Ambit Stochastics*, a new field of mathematical stochastics that has its origin in the study of turbulence, but is in fact of broad applicability in science, technology and finance, in relation to modelling of spatio-temporal processes. For further information, visit: https://sites.google.com/site/ahoinet/

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Abstracts of Talks

On Φ-variation of Hermite processes

Andreas Basse-O'Connor AARHUS UNIVERSITY

The Φ-variation of a stochastic process may be viewed as a measurement of its smoothness, and it plays an important role in integration theory, rough paths theory and Fourier analysis. The special cases of bounded variation and bounded p-variation were introduced by Jordan and Wiener respectively, and the general definition of Φ-variation goes back to Young. A classical result by Lévy states that the sample paths of a Brownian motion are of bounded p-variation if and only if $p > 2$. This result has been improved by Taylor who showed that the "correct" Φ-variation of the Brownian motion is $\Phi(x) = x^2/\log(\log(1/x))$. Furthermore, Dudley and Norvaisa has characterized the correct Φ-variation of the fractional Brownian motion. On the other hand, there does not exists a correct Φ-variation function for a stable non-Gaussian Lévy process. In this talk we will derive the correct Φ-variation of a class of self-similar Gaussian chaos processes called Hermite processes. This class includes the fractional Brownian motion and Rosenblatt process as special cases. Our technique relies on metric entropy methods for stochastic processes with exponential moments. This talk is based on joint work with Michel Weber, Université de Strasbourg.

Volatility Modelling: Beyond the COGARCH

Anita Behme TECHNISCHE UNIVERSITÄT MÜNCHEN

The COGARCH model as introduced 2004 by Klüppelberg et al shows a similar behaviour as the GARCH and captures many properties of financial data. Nevertheless it also has some unwanted properties as for example it is (as the GARCH) not able to reproduce a leverage effect. Also, as in other one-parameter models, jumps of volatility and price in the COGARCH model have a deterministic relation. In this talk we shortly recall the COGARCH model and discuss how one can improve it by allowing some asymmetry and using superposition techniques.

Simulation of stochastic Volterra equations driven by space-time Lévy noise

Carsten Chong TECHNISCHE UNIVERSITÄT MÜNCHEN

In this talk we investigate two numerical schemes for the simulation of stochastic Volterra equations driven by space-time Lévy noise of pure-jump type. The first one is based on truncating the small jumps of the noise, while the second one relies on series representation techniques for infinitely divisible random variables. Under reasonable assumptions, we prove for both methods L^p- and almost sure convergence

of the approximations to the true solution of the Volterra equation. We give explicit convergence rates in terms of the Volterra kernel and the characteristics of the noise. A simulation study visualizes the most important path properties of the investigated processes. This talk is based on joint work with Bohan Chen and Claudia Klüppelberg.

Self-exciting jump processes: Estimation and asymptotic theory

Heidar Eyjolfsson University of Bergen

We present work in progress on self-exciting jump processes. We discuss a class of mean-reverting, and self-exciting continuous-time jump processes. Our class of processes can have stochastic jump sizes in the intensity process, as opposed to just constant jump sizes, which is more common in the literature. We give a short overview, with references, of the development of such processes, discuss maximum likelihood estimation, and put them into context with processes that have been proposed recently. We moreover study Markovian aspects, and present some limit theorems.

Nonparametric estimation of kernel functions of Brownian semi-stationary processes

Shin Kanaya Aarhus University

This paper considers nonparametric estimation of Brownian semi-stationary processes (BSS; also known as continuous-time moving-average processes). We establish fully nonparametric identification of the so-called kernel functions of the processes through their covariance functions. This identification result allows us to propose a nonparametric series/sieve estimator of the kernel functions. We also investigate asymptotic properties of the proposed estimator, as well as its finite-sample properties through a simulation study. This talk is based on joint work with Asger Lunde and Orimar Sauri.

Hard-core thinnings of germ–grain models with power-law grain sizes

Lasse Leskelä Aalto University

Random sets with long-range dependence can be generated using a Boolean model with power-law grain sizes. This talk focuses on thinnings of such Boolean models which have the hard-core property that no grains overlap in the resulting germ–grain model. A fundamental question is whether long-range dependence is preserved under such thinnings. To answer this question we study four Matérn-type thinnings of a Poisson germ–grain model where the grains are spheres with a regularly varying size distribution. It turns out that a thinning which favors large grains preserves the slow correlation decay of the original model, whereas a thinning which favors

small grains does not. The most interesting finding concerns the case where only disjoint grains are retained, which corresponds to the Matérn type I thinning. In the resulting germ–grain model, typical grains have exponentially small sizes, but rather surprisingly, the long-range dependence property is still present. As a byproduct, we obtain new mechanisms for generating homogeneous and isotropic random point configurations having a power-law correlation decay. This talk is based on a paper with the same title (Adv. Appl. Probab. 2013, arXiv:1204.1208), joint work with Mikko Kuronen (U Jyväskylä).

Gaussian Phase Transitions for Conic Intrinsic Volumes

Ivan Nourdin University of Luxembourg

Recent empirical research indicates that many convex optimization problems with random constraints exhibit a phase transition as the number of those constraints increases. In this talk, I will explicitly connect this phase transition with the asymptotic Gaussian fluctuations of the intrinsic volumes of the descent cone that is canonically attached to the convex optimization problem at hand. My presentation will revolve around the many remarkable properties that Gaussian vectors possess; as such, it will provide ideas and tools that might be of interest for researchers interested in the analysis of tempo-spatial models.

A change of measure preserving the affine structure in the BNS model for commodity markets

Salvador Ortiz-Latorre University of Oslo

In this talk we consider a commodity spot price dynamics given by an Ornstein-Uhlenbeck process with Barndorff-Nielsen and Shephard stochastic volatility. We price forwards using a class of pricing measures that simultaneously allow for change of level and speed in the mean reversion of both the price and the volatility, while preserving the affine structure of the model. We demonstrate that the theoretical risk premium in our model can provide the typical shapes observed in energy markets. In particular, it allows for a stochastic change of sign and it can attain positive values in the short end of the forward curve and negative in the long end.

Limit theorems for stationary increments Lévy driven moving average processes

Mark Podolskij Aarhus University

In this paper we present some new limit theorems for power variation of kth order increments of stationary increments Levy driven moving average processes. In this infill sampling setting, the asymptotic theory gives very surprising results, which (partially) have no counterpart in the theory of discrete moving average processes. More specifically, we will show that first order limit theorems and the mode of con-

vergence strongly depend on the interplay between the given order of the increments, the considered power $p > 0$, the Blumenthal-Getoor index $\beta \in (0, 2)$ of the driving pure jump Levy process L and the behaviour of the kernel function g at 0 determined by the power α. First order asymptotic theory essentially comprise three cases: stable convergence towards a certain infinitely divisible distribution, an ergodic type limit theorem and convergence in probability towards an integrated random process. We also prove the second order limit theorem connected to the ergodic type result. When the driving Lévy motion L is a symmetric β-stable process we may obtain two different limits: a central limit theorem and convergence in distribution towards a symmetric $(1 - \alpha)\beta$-stable random variable.

Functional limit theorems for generalized variations of the fractional Brownian sheet

Anthony Réveillac INSA de Toulouse

In this talk we present functional central and non-central limit theorems for generalized variations of d-dimensional fractional Brownian sheets (fBs). In the former case we obtain an independent fBs as a limit, whereas in the latter case we prove that the limit is given by an Hermite sheet driven by the same noise as the original fBs. These results apply to the study of power variations of the fBs for which we derive functional limit theorems. This talk is based on a joint work with Mikko S. Pakkanen.

Cylindrical Lévy processes

Markus Riedle King's College London

The objective of this talk is the introduction of cylindrical Lévy processes and their stochastic integrals in Hilbert spaces.

The degree of freedom of models in infinite dimensions is often reflected by the request that each mode along a dimension is independently perturbed by the noise. In the Gaussian setting, this leads to the *cylindrical Wiener process* including from a model point of view the very important possibility to model a Gaussian noise in both time and space in a great flexibility (space-time white noise). Up to very recently, there has been no analogue for Lévy processes.

Based on the classical theory of cylindrical processes and cylindrical measures we introduce *cylindrical Lévy processes* as a natural generalisation of cylindrical Wiener processes. We illustrate our abstract approach by several specific examples of cylindrical Lévy processes. We continue to characterise the distribution of cylindrical Lévy processes by a cylindrical version of the Lévy-Khintchine formula. The last part of the talk is devoted to explain the difficulty to introduce a stochastic integral with respect to cylindrical Lévy processes, and how we could solve this problem.

(parts of this talk are based on joint work with D. Applebaum or A. Jakubowski)

Towards Estimation of Lévy driven CARMA processes from high-frequency non-equidistant observations

Robert Stelzer Ulm University

In this talk we shall first review the definition and important properties of Lévy-driven CARMA processes focusing on second order properties and spectral representations. We shall also briefly recall which estimation techniques based on (high-frequency) equidistant observations have been investigated so far.

Thereafter we are going to present some results concerning estimation in the frequency domain based on continuous observations. The idea is to estimate the spectral density of CARMA processes by the so called truncated Fourier transform. This approach is then generalized to a high-frequency setting with non-equidistant observations using appropriate results on numerical integration procedures.

This talk is based on joint work with Zywilla Fechner.

Integration w.r.t. time-changed volatility modulated Volterra processes

Benedykt Szozda Aarhus University

We develop the theory of stochastic integration with respect to volatility modulated Volterra processes driven by time-changed Lévy processes. We extend recent results that used Volterra processes driven by either a Brownian motion or a square-integrable, zero-mean pure-jump Lévy processes with stochastic modulation of the amplitude of the volatility. Our approach allows for more general driving noises like, for example, α-stable processes. From the modelling perspective, we allow for independent modulation of the amplitude and the intensity of the stochastic volatility obtaining a more flexible modelling framework. This is joint work with Ole E. Barndorff-Nielsen and Fred Espen Benth.

Multivariate limit theorems involving short-range and long-range dependence

Murad S. Taqqu Boston University

Consider a vector of multilinear polynomial-form processes with either short or long memory components. The components have possibly different coefficients but same noise elements. We study the limit of the normalized partial sums of the vector and identify the independent components. These results are extended to generalized Hermite processes.

Positive semi-definite estimators via subsampling in high frequency data

Bezirgen Veliyev AARHUS UNIVERSITY

We construct positive semi-definite estimators for asymptotic conditional variance in high-frequency data framework. Our approach is based on a novel idea of subsampling in high frequency data. We derive a sharp convergence rate for this estimator. We also discuss connections to the testing for the presence of jumps.

Abstracts of Posters

Estimation of integer-valued trawl processes

Mikkel Bennedsen AARHUS UNIVERSITY

We present a simple composite likelihood framework to estimate a general integer-valued trawl process and investigate the performance of the estimations using a wide variety of trawl functions. We then extend the model to include stochastic volatility, through a stochastic time-change, and show how one can estimate the resulting process using particle filter techniques.

A law of large numbers for the 2-dimensional Brownian semistationary process with stochastic correlation

Andrea Granelli IMPERIAL COLLEGE LONDON

The *Brownian semistationary process* is a Gaussian process without independent increments, which has recently been used in the context of turbulence modelling, as a model for the field of the velocity vectors in a turbulent flow.

We work in a fixed time horizon and sample our process at a discrete set of observations, and we let the number of observations go to infinity. Thus we are in the so-called framework of *infill asymptotics*. It is well known that if X is a semimartingale, then along a sequence of partitions of $[0, t]$, whose mesh goes to zero, the sum of the squared increments of X converges in probability to the quadratic variation of the semimartingale X accumulated up to time t, which is a.s. finite, for all t.

When X lies outside the semimartingale class, such general theorems fail to hold, a counterexample being the fractional Brownian motion. The Brownian semistationary processes, in general, is not a semimartingale. Nevertheless, there are results in the literature that show that the realised quadratic variation process still converges to a finite limit.

In this work we aim to prove a corresponding result for the quadratic covariation of two Brownian semistationary processes, showing the convergence of the realised covariation process in an appropriate sense. We will use our results in order to estimate the stochastic correlation between two assets.

Power variation limits for Lévy driven moving average processes

Claudio Heinrich AARHUS UNIVERSITY

Developing a deep understanding of (purely temporal) Lévy driven moving average

processes of the type

$$X_t = \int_{-\infty}^{t} g(t-s) b_s dL_s,$$

where g is a deterministic kernel and b is predictable, is certainly one fundamental goal of ambit theory nowadays. We consider symmetric β-stable pure jump Lévy motions as integrators and derive limits for the power variation

$$V(p)_n = \sum_{i=1}^{n} |X_{i/n} - X_{(i-1)/n}|^p$$

for $n \to \infty$.

Modelling turbulent time series by BSS-processes

Ulises Márquez Aarhus University

Brownian semi-stationary processes have been proposed as a class of stochastic models for time series of the turbulent velocity field. We show, by detailed comparison, that these processes are able to reproduce the main characteristics of turbulent data. Furthermore, we present an algorithm that allows to estimate the model parameters from second and third order statistics. As an application we reproduce a turbulent time series measured in a helium jet flow.

From Time to Space-time: Tempo-spatial Ornstein-Uhlenbeck Processes

Michele Nguyen Imperial College London

Tempo-spatial modelling is an emerging research area which combines techniques from time series, spatial statistics and stochastic processes. Our project contributes to this line of research by developing the theory, simulation and inference methods for a tempo-spatial extension of the Lévy-driven Ornstein-Uhlenbeck (OU) process. We show that the new process shares many properties with the original OU process such as stationarity and ergodicity. In addition, it offers flexible spatial autocorrelation structures which are useful for modelling. As the process is defined in terms of a stochastic integral, we developed two discrete convolution algorithms to generate data from the model. These algorithms were compared through simulation experiments where the model parameters were estimated using a tempo-spatial moments-matching method.

A latent trawl process model for extreme values

Ragnhild Noven Imperial College London

Trawl processes are a class of stationary, continuous-time stochastic processes driven by a Lévy random measure, which allow for a flexible autocorrelation structure. We explore a model that is based on a mixture decomposition of the generalised

Pareto distribution, which can be used to model exceedances over a threshold. The resulting hierarchical model uses a trawl process as a latent component, which allows for temporal correlation in the observed exceedances. We discuss properties of the model in the context of making inference based on observed exceedances.

Invertibility of infinitely divisible continuous time moving average processes

Orimar Sauri AARHUS UNIVERSITY

In this poster I will present sufficient conditions for the invertibility of continuous time moving average processes driven by a Lévy processes. Such conditions are entirely determinated by the kernel and no moments in the background driving Lévy proces (BDLP) are required. Moreover, these conditions are analogous to the discrete time case and equivalent in the square integrable case. Using this, we show that the law of the process is uniquely determined by the law of the BDLP.

Limit Theorems for U- and V-statistics of Ito Semimartingales

Christian Schmidt AARHUS UNIVERSITY

We study the asymptotic theory for U- and V-statistics of high frequency observations of Ito semimartingales of the form

$$X_t = x + \int_0^t a_s ds + \int_0^t \sigma_s dW_s + J_t, \quad t \in [0,T].$$

In the case of a vanishing jump part J_t, we prove, based on empirical process methods, uniform convergence in probability and show a functional stable central limit theorem for the standardized version of a U-statistic of some order d, i.e. for

$$U(H)_t^n = \binom{n}{d}^{-1} \sum_{1 \le i_1 < \ldots < i_d \le \lfloor nt \rfloor} H(\sqrt{n}\Delta_{i_1}^n X, \ldots, \sqrt{n}\Delta_{i_d}^n X),$$

$$(\Delta_i^n X = X_{i/n} - X_{(i-1)/n}).$$

If jumps are included, we consider a similar types of statistics and give laws of large numbers and central limit theorems.

Special Event

On Tuesday, 24 February 2015 at 17:00, shortly after the workshop, the Imperial College Statistics Seminar features a talk by a distinguished speaker, Prof Bernard Silverman FRS. The workshop participants are encouraged to attend this special seminar talk, which takes place in the Clore Lecture Theatre of Huxley Building (180 Queen's Gate) and is followed by a drinks reception.

Science and Mathematics in the Home Office

Prof Bernard Silverman FRS HOME OFFICE

I will describe my role and work as Chief Scientific Adviser in the Home Office, and describe a range of examples where mathematics and science have a demonstrable impact on policy, with a focus on areas where statistical thinking and expertise has been useful. In Forensic Science alone, these range from Protection of Freedoms legislation about the retention of DNA profiles to an evaluation of the risks of new DNA profiling protocols. My major illustrative example, however, will be the novel use of multiple systems estimation to gain insight into the scale of Modern Slavery in the UK, the way this has fed into the Governments Modern Slavery Strategy, and the wider science/policy issues this work presented.

Participants

Andreas Basse-O'Connor (Aarhus University, Denmark)
Anita Behme (Technische Universität München, Germany)
Mikkel Bennedsen (Aarhus University, Denmark)
Laurent Chaminade (Imperial College Press, UK)
Jean-François Chassagneux (Imperial College London, UK)
Carsten Chong (Technische Universität München, Germany)
Heidar Eyjolfsson (University of Bergen, Norway)
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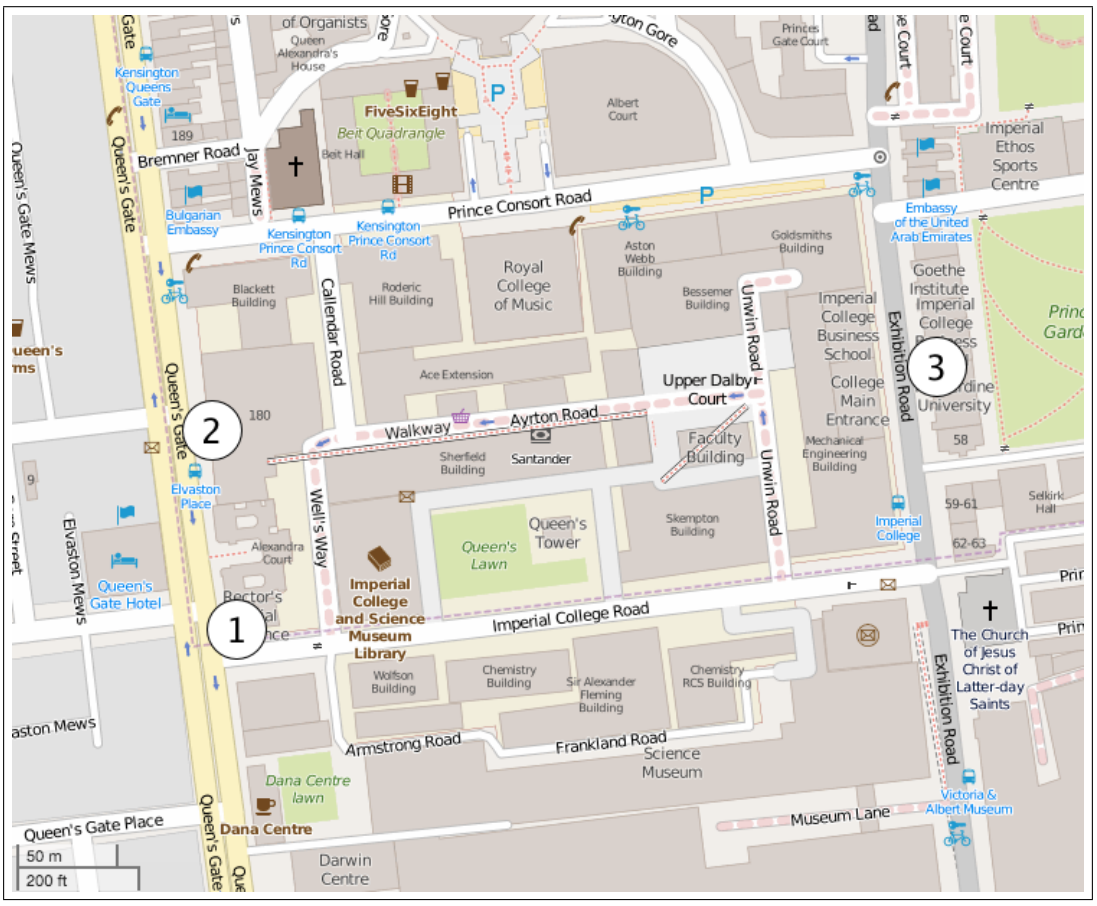
Benedykt Szozda (Aarhus University, Denmark)
Murad S. Taqqu (Boston University, USA)
Bezirgen Veliyev (Aarhus University, Denmark)
Almut Veraart (Imperial College London, UK)

Map of the South Kensington Campus



1. Workshop venue (170 Queen's Gate)
2. Huxley Building/Department of Mathematics (180 Queen's Gate)
3. Ognisko Restaurant (55 Prince's Gate, Exhibition Road)